Short-time memories in a network with randomly distributed connections

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Coupled map lattices are able to store short-term memories when an external periodic input is applied. We consider short-term memory formation in networks with both regular (nearest-neighbor) and randomly chosen connections. The regimes under which single or multiple memorized patterns are stored are studied in terms of the coupling and nonlinear parameters of the network.

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Coppersmith and co-workers have shown the existence of a novel collective effect in lattices of coupled dynamical systems forced by external inputs: the system "remembers" the inputs it receives for a transient period, but it coarsens and eventually "forgets" those inputs [1]. This effect has been called self-organized *short-time memory* and observed in a sliding charge density wave experiment in $NbSe_3$ [1]. A simple model for this situation consists of an overdamped chain of masses connected by linear springs, one of its ends being clamped and the other subjected to a periodic sequence of impulses [2]. The short-time memories are the synchronized response of the chain to the repeated train of driving pulses: multiple memories (i.e., values of the input strength which are echoed by the chain) are encoded during a transient time period.

The mass-spring chain with periodic impulses can be mathematically described by a coupled map lattice (CML), in which both space and time are discrete variables, but with a continuous state variable [3]. A CML is composed basically of a local dynamical unit that undergoes a discrete temporal evolution, interacting with other units through a given coupling prescription. The state variable may be interpreted as the position of each mass inside a deep potential well with respect to its equilibrium value. The coupling prescription is given by the net Hookean force provided by the springs connecting nearest-neighbor masses, which also accounts for the linear dynamical process at each discrete position.

Let $x_n^{(i)}$ be the particle position at the ith potential well, where $i=1,2,\ldots,N$ for a one-dimensional chain at discrete time $n=0,1,2,\ldots$. We consider linear springs of force constant K and an external input signal consisting of impulsive kicks applied at discrete times with strength $(1+A_n)$. The sequence of A_n is assumed to have a given periodicity in time and it constitutes the pattern that the network is supposed to memorize. This memory is short termed because it lasts only while the inputs are being applied, unlike a Hopfield type of memory which minimizes an energy function in neural networks [4]. The corresponding CML is given by [1,2]

$$x_{n+1}^{(i)} = x_n^{(i)} + \mathrm{int}\{K[x_n^{(i+1)} - 2x_n^{(i)} + x_n^{(i-1)}] - (1+A_n)\}, \quad (1)$$

where $\mathrm{int}\{z\}$ is the largest integer less than or equal to z. For linear springs the entire sequence of inputs is retrieved during a transient period and just a few inputs are memorized for long times. This limitation has been proved to be circumvented by the use of slightly nonlinear springs: the CML (1) has to be modified by replacing the state variable x inside the int prescription by $f(x)=x+Rx^2$ with a small parameter $R \ll 1$ [5]. In this case multiple memories can be encoded for long times, an effect also obtained using chains of linear springs with small noise [6].

Although short-time memories were first described in overdamped mass-spring chains, they have been shown to occur in other systems like inductively coupled circuits [7], and lattices with nonlocal couplings where the particles interact with all other particles, their mutual interaction decaying with the lattice distance in a power-law fashion [8]. The common feature of these systems is that all are represented by regular lattices, for which there is a kind of translational symmetry of the coupling term.

However, there is a growing interest in the study of random lattices, where the connections between sites (not necessarily close to each other) are randomly chosen according to a specified probability distribution [9]. Recent investigations on small-world networks have raised the need for lattice models with both regular and random properties [10,11]. In the Watts-Strogatz lattices some of the regular couplings (between nearest and next-to-nearest neighbors) are rewired and connected to randomly chosen sites [11]. The Newman-Watts models introduce such random shortcuts without rewiring [10,12]. This Brief Report addresses the existence of short-time memories also in lattices with both regular and random couplings obtained from the Newman-Watts procedure.

We further modify the term within brackets in the coupling prescription of the CML (1) to introduce the contribution of the nonlocal random shortcuts:

$$f(x_n^{(i+1)}) - 2f(x_n^{(i)}) + f(x_n^{(i-1)}) + \mathcal{M}_n^{(i)}, \quad (2)$$

where $\mathcal{M}_n^{(i)}=\Sigma_j[f(x_n^{(j)})-f(x_n^{(i)})]I_{ij}$, in which I_{ij} is an adjacency matrix with entries 1 and 0 if the sites i and j have or do not have a shortcut connecting them, respectively.

The adjacency matrix is symmetric ($I_{ij}=I_{ji}$) and the nonzero elements are randomly chosen according to a uniform probability P, which turns out to be the ratio of the number of nonlocal shortcuts N_1 and the total number $N_T=(N^2-3N+2)/2$ of connections, excluding self-interactions and interactions with nearest neighbors. We used a random number

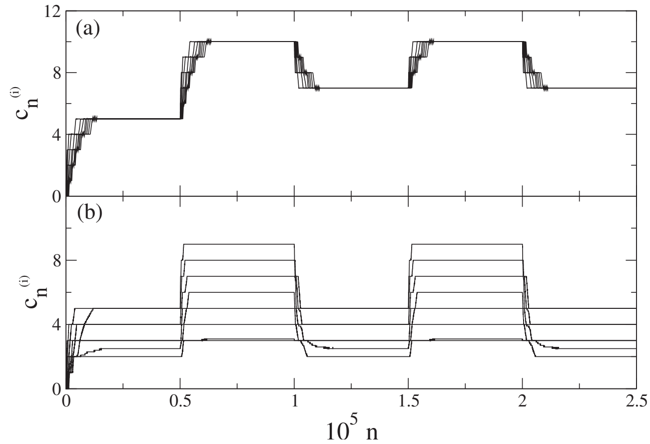

FIG. 1. Curvature variables for a lattice of $N=10$ locally coupled maps of the form (1) with $K=0.01$. The kick amplitude was switched from $A_n=5$ to 10 in regular time intervals. $R=$(a) 0 and (b) 10^{-8}.

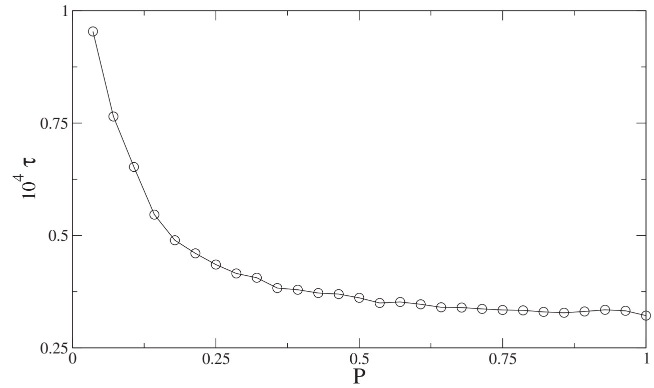

FIG. 2. Average time taken for the lattice to achieve a stationary short-term memory as a function of the uniform probability P used to obtain the adjacency matrix of the lattice with $N=10$ sites, and the following parameters: $R=0$, $A_n=10$, $K=0.01$. We consider 50 different random realizations of the adjacency matrix.

generator with uniform probability that gives a pseudorandom number $0 \leqslant r_n \leqslant 1$. After choosing the number of random shortcuts N_1 we obtain the position (i,j) of the first nonzero matrix element in the following way: i is the nearest integer less than or equal to $r_n N_1$; for j we use the same procedure. However, if by chance $j=i$ or $i\pm 1$, we discard this run and repeat the procedure until all N_1 nonzero matrix elements are chosen.

For the sake of short-term memory formation it is necessary that one of the ends of the chain is kept nailed, whereas the end where the inputs are applied must be free. This makes for mixed boundary conditions: $x_n^{(1)}=0$, $x_n^{(N+1)}=x_n^{(N)}$. In order to satisfy this requirement, the elements of the adjacency matrix belonging to either the last row or column are equal to zero, since the site at the free end is coupled only with the site for which $i=N-1$.

The memorized patterns are retrieved from the discrete curvature variables, defined as

$$c_n^{(i)} = K[f(x_n^{(i+1)}) - 2f(x_n^{(i)}) + f(x_n^{(i-1)}) + \mathcal{M}_n^{(i)}], \qquad (3)$$

which are nothing but the coupling terms for each site of the CML (1) without the prescription of taking the integer part. The input amplitudes A_n are considered memorized by the lattice when the curvature variables for a number of sites take on a constant value for a given time. Once the inputs cease to be applied, the memories disappear very fast due to the strong dissipative character of the lattice dynamics. We assume that the external input is continuously being applied to the chain after an initial time, which is also used as the reference for counting the duration of the memorized patterns. Hence transient and stationary are terms used with respect to this initial time, assuming that the inputs never cease to be applied.

We observe the formation of transient and stationary memories for $N=10$ maps in a one-dimensional lattice with local (nearest-neighbor) couplings of the form given by Eq. (1). We choose an input train with amplitudes alternating between $A_n=5$ and 10 at constant time intervals of duration 0.5×10^5. The time evolution of the curvature variables for *all* lattice sites is depicted in Fig. 1(a) in the case of linear maps, for which there is a single stationary memorized value, which is equal to either A_n or $2+A_n$.

The occurrence of stationary memorized values for the curvature variables is a consequence of our having a fixed point in the dynamics of the CML. From Eq. (1), the condition for having a fixed point x^* implies the existence of a single memorized value c^* of the curvature variable given by $\mathrm{int}(c^*-1-A_n)=0$. We thus have that $-1<c^*-1-A_n<1$, which gives us two inequalities to be satisfied if a memorized value is possible: $c^*<2+A_n$ or $c^*>A_n$. In fact, according to Fig. 1(a), we apply $A_n=5$ during the first 0.5×10^5 time instants, and the curvature variable increases from zero to $c^*=5$. During the next 0.5×10^5 time instants the amplitude changes to $A_n=10$ and the curvature variable increases to match this value. However, during the following 0.5×10^5 instants the amplitude switches back to 5 and the curvature variables decrease to settle down at $c^*=7$, as predicted.

Stationary multiple memories are possible, however, if a slightly nonlinear term is added [Fig. 1(b)]. We emphasize that the latter result does not come from an insufficiently long observation time: as shown in a previous paper the multiple memories represent stationary solutions of the dynamical system (1) [5]. We claim that the memorized patterns observed in purely regular lattices can also be possible if random connections are added to the lattice, with some advantages with respect to regular lattices.

A relevant quantity here is the time τ it takes for the lattice to achieve a stationary memorized pattern. We observe that, in the case of multiple memories, this time is different for each memorized value. As can be seen in Fig. 1(a), the approach to the stationary pattern is done in a staircase way. The sites whose curvature variables are farther from the common stationary value take more time to achieve this state because they have to take on all the intermediate values. Moreover, even this time also depends on the particular realization of the randomly chosen entries of the adjacency matrix. Hence a statistically reasonable quantity is the aver-

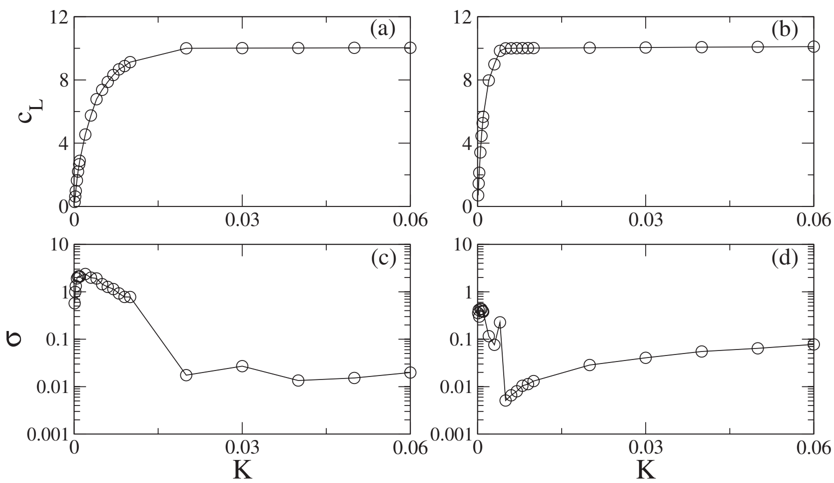

FIG. 3. Average value of the curvature variable as a function of the coupling strength with P=(a) 0 and (b) 0.5. Standard deviation for P=(c) 0 and (d) 0.5. Other parameters are N=10, A_n=10, and R=10^{-9}.

age time it takes for all sites to achieve a stationary memory, considering different random realizations for the adjacency matrix.

In Fig. 2 we depict the average transient time τ as a function of the uniform probability P used to obtain each realization of the adjacency matrix. The fact that τ decreases with the probability can be qualitatively understood: the higher the value of P is the more randomly chosen shortcuts are found in the lattice. Since the achievement of a stationary memory is essentially a coupling effect, it is not surprising that the time it takes to get such a memory diminishes with increasing P.

When a lattice stores multiple stationary memories, each of them characterized by a given value of the curvature variable, a useful quantity to describe this effect is the average value of the curvature variable c_L as well as its standard deviation with respect to that mean, σ. The averages are taken over both the entire lattice and also by using different initial realizations of the adjacency matrix. When the lattice presents one permanent memory, the curvature average c_L is equal to the external input A_n, with no variance at all (σ_L =0), whereas it takes on a value less than A_n when multiple memories are present, and with positive variance ($\sigma_L > 0$).

The dependence of the average curvature value and its variance with the coupling strength is depicted in Fig. 3 for different values of the probability with which the adjacency matrix was built. In all cases the average value saturates at c_L=10 (the same value of the external input) for strong enough coupling, which represents a measure of the capacity of the lattice to store a given memorized pattern. This saturation occurs with different values of the coupling strength, from $K \approx 2$ for a lattice without shortcuts [Fig. 3(a)] to $K \approx 0.5$ for a lattice with shortcuts [Fig. 3(b)]. Moreover, the variance next goes to zero at the saturation [Fig. 3(c)] and increases afterward when $P \neq 0$ [Fig. 3(d)].

The influence of the lattice size N on the average curvature variable and its variance is depicted in Fig. 4. The value of c_L decreases and σ increases with N for fixed values of A_n, K, and R for both small P [Figs. 4(a) and 4(c)] and large P [Figs. 4(b) and 4(d)], without essential differences. In both cases we see that increase of the lattice size has a deleterious effect on the process of memory formation, since the curvature variable clusters at small values with likewise small dispersion, which may make the process nonfeasible for practical applications.

The formation of multiple stationary memories is of potential use in, e.g., schemes for coding information in graphic (pixel) matrices [5,7]. Hence it is important to know for what parameter choices the lattice will store those multiple memories. We set a large time interval T and follow the curvature variable in order to find whether or not we have stationary memories for a given parameter set. We analyze the depen-

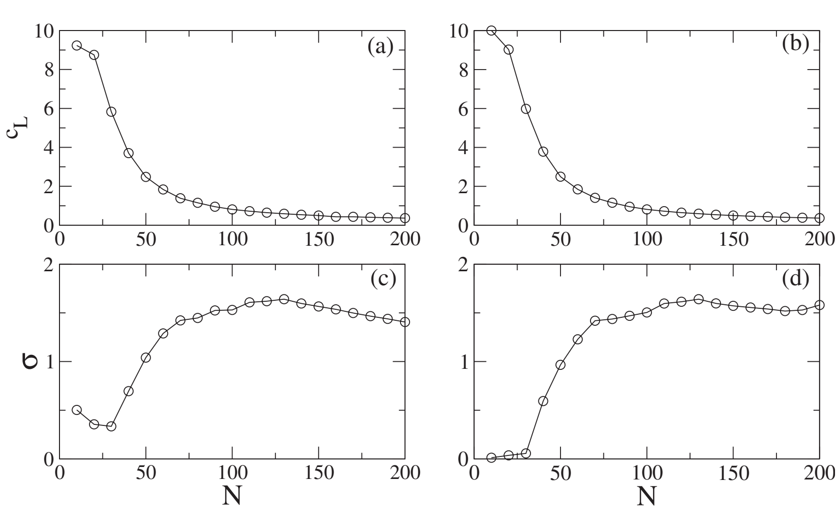

FIG. 4. Average value of the curvature variable as a function of the lattice size N for P=(a) 0.1 and (b) 0.7. (c) and (d) are the standard deviations corresponding to (a) and (b), respectively. Other parameters are A_n=10, K=0.01, and R=3.5×10^{-9}.

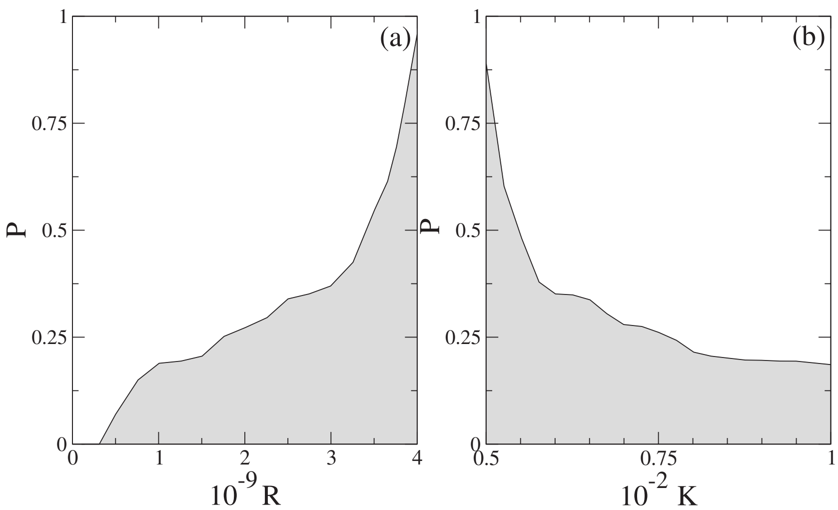

FIG. 5. Parameter planes for the lattice: the white regions correspond to only one permanent memory and the gray regions to the formation of multiple memories. (a) $P\times R$ with $K=0.01$. (b) $P\times K$ parameter plane with $R=10^{-9}$. Other parameters are $A_n=10$ and $N=10$.

dence of the probability P on the nonlinearity parameter R in Fig. 5(a), where the white region corresponds to values of P and R such that the lattice presents one permanent memory, the gray region corresponding to multiple memories. The regular lattice case $P=0$ was formerly claimed to exhibit such memories only for $R\neq 0$.

However, in Fig. 5(a), there appears an interval of values ($R<3\times 10^{-10}$) without multiple memories. Moreover, with increase of the probability P there are intervals for which there is only one stationary memory, regardless of the duration T. In the limiting case of $P=1$ there are plenty of shortcuts and the lattice exhibits no multiple memories at all. Hence the random shortcuts act collectively as inhibitors of short-time memory formation. Essentially the same conclusion holds if we consider as variable parameters the probability P and the nonlinearity R [Fig. 5(b)].

In conclusion, we explored some aspects of the spatiotemporal dynamics displayed by a coupled map lattice with random interactions, using an adjacency matrix with entries chosen randomly with a given (uniform) probability which can be varied in order to simulate networks with a variable number of shortcuts per site. The memories are short termed since they echo an external periodic input as long as it is still being applied to the system. The memories are retrieved from computation of the so-called curvature variable. Our results point out two possible, and qualitatively different, scenarios, single or multiple stationary memories, depending on the parameters of the coupled map lattice. The latter scenario is more interesting for possible applications like coding of messages or graphical matrices. The existence of some nonlinearity is a desirable (though not necessary) condition for multiple memory storage. However, the presence of random shortcuts turns out to be an inhibitory factor for this effect, since the fraction of parameters for which multiple memories exist diminish with increasing probability of random shortcuts. In spite of this, the time it takes for stationary memories diminish with this probability. We also show that better results in terms of memory formation can be obtained with relatively small lattices (fewer than 50 sites), in comparison with larger ones. This makes lattices with both regular and random connections appealing to store graphical pixel matrices required for, e.g. image compression and storage.

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